Trigonometry Practice Problems for Precalculus and Calculus

1. $\frac{\pi}{24}$ radians corresponds to ___________ degrees.

2. $240°$ corresponds to ___________ radians.

3. On a circle of radius 2, a (central) angle measuring $\frac{\pi}{6}$ radians spans an arc of length ___________.

4. Find an angle θ, with $0° < \theta < 360°$ with the same sine as $240°$.

5. True or False? $\sin(x + 2\pi) = \sin(x)$ for all x.

6. True or False? $\sin\left(x - \frac{\pi}{2}\right) = \cos(x)$ for all x.

7. True or False? $\cos(3)$ is a positive number.

8. Find all values of t that solve the equation $2\sin(t) - 1 = 0$ when $0 \le t \le 2\pi$.

9. What is the period of the function $f(x) = \cos\left(\frac{\pi}{8}x\right)$?

10. What is the amplitude of the function $f(x) = 10\sin(2x)$?

11. Write the following expression in terms of sines and cosines and simplify: $\sec(x)\tan(x) + \tan^2(x)$

12. Identify the domain and range of the functions $f(x) = \cos^{-1}(x)$, $g(x) = \sin^{-1}(x)$, and $h(x) = \tan^{-1}(x)$.

13. Find the exact values of the following quantities.

    (a) $\tan(\frac{\pi}{3}) =$

    (b) $\cos^{-1}(\frac{\sqrt{3}}{2}) =$

    (c) $\arcsin(-1) =$

    (d) $\arctan(1) =$

    (e) $\sin^{-1}(\sin(2\pi)) =$

14. Assume $0 < x < 1$. Simplify the expression $\sin(\cos^{-1}(x))$.

Solutions

1. To find what an angle of $\frac{\pi}{24}$ radians is in degrees, we must multiply by the conversion factor of $\frac{180}{\pi}$ degrees per radian to get $\frac{\pi}{24} \cdot \frac{180}{\pi} = \frac{180}{24} = \frac{90}{12} = \frac{45}{6} = \frac{15}{2} = 7.5$ degrees.

2. To find what an angle of $240°$ is in radians, we must we must multiply by the conversion factor of $\frac{\pi}{180}$ radians per degree to get $240 \cdot \frac{\pi}{180} = \frac{4\pi}{3}$ radians.

3. If s is the arc length spanned by a (central) angle of θ radians on a circle of radius r, then $s = r\theta$. So, in our case, $s = 2 \cdot \frac{\pi}{6} = \frac{\pi}{3}$.

4. Given the unit circle and an angle θ measured counterclockwise around the unit circle starting from the point $(1, 0)$, we know that the sine of this angle is the second coordinate of the point of intersection of the second ray of the angle with the unit circle (if this is confusing, reread it and draw a picture). For example, when you type $\sin(240°)$ on your calculator, you will get an answer of about -0.866 (the true answer is actually $-\frac{\sqrt{3}}{2}$). If you draw an accurate picture, you will see that this is reasonable.

   To find another angle with the same sine value, we must find another angle with the same second coordinate for the point of intersection. After looking at your figure further, you should be able to convince yourself that $\theta = 300°$ is just such another angle. In addition, if you take its sine, you will see that again you get $-\frac{\sqrt{3}}{2}$. Also note that $0° < 300° < 360°$ as required.

   Therefore, $\theta = 300°$ is the answer.

5. The equation $\sin(x+2\pi)=\sin(x)$ is TRUE for all x. This is an *identity* (because it is true for all x). The reason is based on the definition of the sine function in terms of the unit circle (if the angle increases by 2π radians, you will be back where you started...by the way, it is implicit that we are using radians here). Thought of another way, when you shift the graph of the sine function to the left by 2π units, you get the same graph. The sine function is periodic with period 2π.

6. FALSE. The equation $\sin\left(x-\frac{\pi}{2}\right)=\cos(x)$ is NOT true for all x (though it is true for some values of x, like $x=\frac{\pi}{2}$). The functions on the left and right hand side are not the same function.

   The following two statements *are* true:

$$\sin\left(x-\frac{\pi}{2}\right)=-\cos(x) \text{ for all } x \text{ and } \sin\left(x+\frac{\pi}{2}\right)=\cos(x) \text{ for all } x.$$

   Now you think about why (think about the graphs or think about the unit circle).

7. FALSE. $\cos(3)$ is a *negative* number (it is assumed that the 3 is the *radian* measure of an angle...if we had meant 3 degrees, we would have written $3°$). This problem *is* possible to answer without a calculator. The number 3 is just barely less than π, so an angle of 3 radians will have a degree measure just barely less than $180°$ (in particular, its degree measure will also be bigger than $90°$). This means that the point of intersection of the second ray of the angle with the unit circle will be in the second quadrant. Since the cosine is the first coordinate of this point, it will be negative.

8. We take the equation $2\sin(t)-1=0$ and add 1 to both sides to obtain $2\sin(t)=1$. Then we divide both sides by 2 to get $\sin(t)=\frac{1}{2}$. Thus, we are looking for all values of t between 0 and 2π such that $\sin(t)=\frac{1}{2}$. A little bit of remembering trigonometry facts and thinking about the unit circle (see the problems above) reveal that $t=\frac{\pi}{6}$ and $t=\frac{5\pi}{6}$ are the two solutions.

9. The function $f(x)=\cos\left(\frac{\pi}{8}x\right)$ has a period of $2\pi/\frac{\pi}{8}=2\pi\cdot\frac{8}{\pi}=16$. In general, to find the period of $\sin(Bx)$ or $\cos(Bx)$, you compute $\frac{2\pi}{B}$.

10. The function $f(x)=10\sin(2x)$ has an amplitude of 10.

11. By definition,

$$\sec(x)\tan(x)+\tan^2(x)=\frac{1}{\cos(x)}\cdot\frac{\sin(x)}{\cos(x)}+\frac{\sin^2(x)}{\cos^2(x)}=\frac{\sin(x)(1+\sin(x))}{\cos^2(x)}.$$

   But $\cos^2(x)=1-\sin^2(x)=(1-\sin(x))(1+\sin(x))$, so

$$\frac{\sin(x)(1+\sin(x))}{\cos^2(x)}=\frac{\sin(x)(1+\sin(x))}{(1-\sin(x))(1+\sin(x))}=\frac{\sin(x)}{1-\sin(x)}.$$

12. Via memorization, the domain of $\sin^{-1}(x)$ and $\cos^{-1}(x)$ are the same set, that is, the interval $[-1,1]$. The range of $\sin^{-1}(x)$ is the interval $[-\frac{\pi}{2},\frac{\pi}{2}]$. The range of $\cos^{-1}(x)$ is $[0,\pi]$. The domain of $\tan^{-1}(x)$ is the set of all real numbers $\mathbb{R}$. The range of $\tan^{-1}(x)$ is the (open) interval $(-\frac{\pi}{2},\frac{\pi}{2})$.

13. Via memorization, we know

   (a) $\tan(\frac{\pi}{3})=\frac{\sin(\frac{\pi}{3})}{\cos(\frac{\pi}{3})}=\frac{\sqrt{3}/2}{1/2}=\sqrt{3}$

   (b) $\cos^{-1}(\frac{\sqrt{3}}{2})=\frac{\pi}{6}$

   (c) $\arcsin(-1)=-\frac{\pi}{2}$

   (d) $\arctan(1)=\frac{\pi}{4}$

   (e) $\sin^{-1}(\sin(2\pi))=0$ (NOT 2π!!!...the property $\sin^{-1}(\sin(x))=x$ is only true if $-\frac{\pi}{2}\leq x\leq\frac{\pi}{2}$...think about why.)

14. Since $0 < x < 1$, we can interpret $\cos^{-1}(x)$ as being an acute angle of a right triangle (because we will have $0 < \cos^{-1}(x) < \frac{\pi}{2}$). It is also possible to label the adjacent side to this angle as x and the hypotenuse as 1 (because the cosine of the angle equals x (that is, $\cos(\cos^{-1}(x)) = x$) and the cosine of the angle is the length of the adjacent side divided by the length of the hypotenuse). To find $\sin(\cos^{-1}(x))$, we must compute the length of the opposite side and divide it by the length of the hypotenuse.

    Let y be the length of the opposite side. By the Pythagorean theorem, $x^2 + y^2 = 1^2$ so that $y = \sqrt{1 - x^2}$. Therefore,

$$\sin(\cos^{-1}(x)) = \frac{\sqrt{1 - x^2}}{1} = \sqrt{1 - x^2}.$$

    Comment: actually, this equation is true even if $-1 \leq x \leq 0$ or if $x = 1$.